Euler International Mathematical Institute
Saint Petersburg

October 1-4, 2018
Monday-Thursday 11:30-12:30

Tsuyoshi Kato (Kyoto University)
"Non commutative geometry and gauge theory "

In my series of lectures, I will introduce extensions of gauge theory
by integrating it with non commutative geometry from the view point of fundamental groups.
I will select some of topics among the following lists.

(1) Twisted Donaldson invariant

Abstract: We define a twisted Donaldson's invariant using the Dirac operator twisted by flat connections when the fundamental group of a four manifold is free abelian. We also verify non triviality of the invariant by presenting some exotic pairs which are obtained from them. Then using Connes-Moscovici's index theorem in non commutative geometry, we introduce the construction of twisted Donaldson's invariant when fundamental group is non-abelian. It leads to a canonical question which we would address, whether the twisted invariant vanishes when a four manifold admits a connected sum decomposition. We also include basic subjects on both Yang-Mills theory and non commutative geometry.

(2) Higher Nahm transform in non commutative geometry

Abstract: Anti-self-dual (ASD) connections over a compact four manifold X attain the critical values on Yang-Mill functional. Nahm transform is a correspondence from a vector bundle with a connection on X to another vector bundle with a connection on the Picard torus. In the case of four torus, it transforms ASD to ASD. In this talk we propose a noncommutative geometric version of Nahm transform, which generalise Connes-Yang-Mills functional via higher Dixmier trace.

(3) Higher degree of the covering monopole map
Abstract: I will introduce a monopole map over universal covering spaces of compact four manifolds. In particular we can formulate higher degree of the covering monopole map when the linearized maps are isomorphic. It induces a homomorphism between K theory of group C^* algebras. As an application we propose an aspherical inequality on compact aspherical four manifolds. This presents a stronger version to 10/8 inequality by Furuta, in the aspherical class of four manifolds. This holds for many cases which include some complex surfaces of general type. Technically the construction of the covering monopole map requires non linear estimates in Sobolev spaces and will motivate L^p analysis on non compact manifolds.

Researchers and graduate students are encouraged to participate.