

Structured hybrid models and Hilbert's thirteenth problem

Bernold Fiedler

Institute of Mathematics
Freie Universität Berlin

Abstract

When can a function of several variables be written as a finite composition of functions of fewer variables? This is the essence of Hilbert's problem XIII. The answer by young V.I. Arnold in the form given by Kolmogorov says: always, by functions of two variables, if we require just continuity. The answer by Vitushkin says: almost never, if we require differentiability.

Modeling, for example in chemical engineering, frequently provides input-output relations as a composition of unknown black-box and known white-box functions. In such a hybrid structure, let black boxes have at most d inputs – typically much less than the total number of inputs to the composition network. Assuming sufficient, and at times prohibitive, differentiability we indicate how to uniquely identify all unknown "black-box" functions in a tree-structured network from only d -dimensional data on their composition. This addresses the "curse of dimension" in data analysis, and provides extrapolability. For general, possibly branched, feed-forward hybrid structures, we obtain genericity results on the global identifiability of all black boxes. Results are joint work with Stefan Liebscher, Andreas Schuppert, and others. See also

<http://dynamics.mi.fu-berlin.de/>