

Upward Book Embeddings of st -Graphs

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Based on a joint work with Carla Binucci, Giordano Da Lozzo, Emilio Di Giacomo,

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A k -page book embedding $\langle \pi, \sigma \rangle$ of an undirected graph $G = (V, E)$ consists of a vertex ordering $\pi : V \leftrightarrow \{1, 2, \dots, |V|\}$ and of an assignment $\sigma : E \rightarrow \{1, \dots, k\}$ of the edges of G to one of k sets, called *pages*, so that for any two edges on the same page their end-points do not alternate in π . One can think of a k -page book embedding as of a topological representation of a graph in a k -page book: the vertices are drawn along the spine of the book, the edges are represented by semi-circles on the pages of the book and the requirement is that no two edges on the same page intersect. An analogous notion for directed acyclic graphs, where the spine is oriented and the ordering π is required to be such that for each edge its source is below its sink on the spine, is called k -page upward book embedding (k UBE).

Computing k -page book embeddings of directed and undirected graphs is a widely studied problem, which finds applications in a variety of domains, including VLSI design, fault-tolerant processing, parallel process scheduling, sorting networks, parallel matrix computations, computational origami, and graph drawing.

Regarding the computational complexity of computing k UBEs, already since 1999 it is known that deciding whether a directed acyclic graph has a k UBE is NP-complete even for $k = 6$ [Heath and Pemmaraju, 1999]. In our recent work [Binucci et.a., 2019], we showed that the problem remains NP-complete even for $k \geq 3$. In this talk, I will concentrate our positive results on 2UBEs of planar st -graphs, a wide class of planar directed acyclic graphs. On the algorithmic side, I will present polynomial-time algorithms for testing the existence of 2UBEs of planar st -graphs with branchwidth β and of plane st -graphs whose faces have a special structure. These algorithms run in $O(f(\beta) \cdot n + n^3)$ time and $O(n)$ time, respectively, where f is a singly-exponential function on β . On the combinatorial side, I will present two notable families of plane st -graphs that always admit a 2UBE. I will conclude with a discussion of a fascinating open problem on computing k UBEs of planar st -graphs posted by [Nowakowski and Parker, 1989] already 30 years ago.