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An analytic approach to cardinalities of sumsets

The aim of this study is to understand the nature of structures in Z^d, the presence of which implies that the sumset must be large. The archetype is Freiman's theorem that if a set $A \subset Z$ d is proper d-dimensional, then

$$|A + A| \geq (d + 1)|A| - \binom{d+1}{2}.$$

The assumption on dimension can be expressed as $S_d \subset A$ for a d-dimensional simplex S_d. In general, the induced doubling of a set U is the quantity

$$\inf_{A \supset U} \frac{|A + A|}{|A|};$$

our main aim is to give lower estimates for it and related quantities. Applications for the sum-product problem, related to the work of [BC04], will be the subject of another paper. While our main interest is in Z^d, we shall mostly formulate our results for general, typically torsion-free commutative groups. Since we work with finite sets and a finitely generated torsion-free group is isomorphic to some Z^d, it is not more general, but we rarely need the coordinates. In the first part we work with sets, in the second part we study a weighted version which will be necessary for the proof of the main results. By introducing a weighted analog, we will be able to use tensorization: that is we prove a d-dimensional inequality by induction on dimension alongside a two point inequality. This is a method commonly used in analysis, for instance in the Pr´ekopa-Leindler inequality [Pr´e71] and Beckner's inequality [Bec75]. We discuss this more below, but also invite the reader to the excellent survey paper of Gardner [Gar02].

[BC04] Jean Bourgain and Mei-Chu Chang. On the size of k-fold sum and product sets of integers. Journal of the American Mathematical Society, 17(2):473–497, 2004.

[Bec75] William Beckner. Inequalities in Fourier analysis. PhD thesis, Princeton., 1975. [Gar02] Richard Gardner. The Brunn–Minkowski inequality. Bulletin of the American Mathematical Society, 39(3):355–405, 2002.

[Gar02] Richard Gardner. The Brunn–Minkowski inequality. Bulletin of the American Mathematical Society, 39(3):355–405, 2002.

[Pr´e71] Andr´as Pr´ekopa. Logarithmic concave measures with application to stochastic programming. Acta Scientiarum Mathematicarum, 32:301–316, 1971.

Приглашаются все желающие!