

Bifurcations of multiple attractors in a predator-prey system

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This is a review of most recent results on bifurcation and chaos in a known predator-prey system and thus an update of the papers in the references. The system of n competing predators feeding on the same prey is of the type

$$X'_i = p_i \varphi_i(S) X_i - d_i X_i, \quad i = 1, \dots, n, \quad (1a)$$

$$S' = H(S) - \sum_{i=1}^n q_i \varphi_i(S) X_i, \quad (1b)$$

where the variable S represents the prey and the variables X_i represent the predators. They are, of course, non-negative. The function φ_i is assumed non-decreasing.

We consider the case where

$$H(S) = r S \left(1 - \frac{S}{K}\right), \quad \varphi_i(S) = \frac{S}{S + A_i}, \quad (2)$$

and where the parameters r , K and A_i are positive.

The dynamics in the coordinate planes representing one of the predators and the prey is well known and there is no more than one cycle. The system has no equilibrium, where predators coexist (in non-degenerate cases). But the predators can coexist in a cyclic and complicated way. There exists multiple attractors of cyclic and different chaotic chaos including "spiral-like" chaos. This happens even in cases, where the populations do not become unrealistic low. We present new discovered phenomena and discuss the possible bifurcations of these and contours from where they could develop.

References:

1. Osipov, A, V., Söderbacka, G. *Poincaré map construction for some classic two predators - one prey systems*. Internat. J. Bifur. Chaos Appl. Sci. Engrg 27, BN^o1, 1750116, 9 pp, (2017).
2. Osipov, A, V., Söderbacka, G. *Review of results on a system of type many predators- one prey*, Nonlinear Systems and Their Remarkable Mathematical Structures, pp 520-540, CRC Press, 2018.
3. Söderbacka, G. J., Petrov, A. S. *Review of the behaviour of a many predators- one prey system*. Dinamicheskie Sistemy, 9(37), N 3, pp 273-288, (2019)