St. Petersburg Algebraic Groups Seminar

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Group-like small cancellation theory for rings (joint work with A. Kanel-Belov, E. Plotkin, E. Rips)

Agatha Atkarskaya (Bar Ilan University)

It is well known that small cancellation groups play a crucial role in the solution of long-lasting problems. Namely Burnside problem, Tarski monster problem and so on. In the talk I will present a construction of a similar object for associative rings, that is a small cancellation associative ring. I am going to give a brief overview of small cancellation groups and then explain how the theory works for the case of rings.

In more details, let F be a free group of a finite rank, and k be a field. Let I be an ideal of kF generated as an ideal by a set of generators R. We impose special conditions on R that are similar to small cancellation conditions for groups. We study the quotient algebra kF/I. We prove that kF/I is non-trivial and explicitly construct its linear basis. Moreover, we show that the ideal membership problem for the ideal I is solvable.

It is well-known that finitely presented small cancellation groups are word-hyperbolic. So, our work is an attempt to express an idea of negative curvature for rings. For groups we have a naturally corresponding geometric object, namely, its Cayley graph. For rings we do not have such object, so, we are working using purely combinatorial methods. That is, the relation to geometry is only indirect. Nevertheless, we feel that negative curvature is an important underlying force in our study.

On the one hand, our algorithmic approach can be considered as an extension of Dehn's algorithm, which we have in hyperbolic groups. On the other hand, the circle of ideas that we are using in our proof has a very clear analogy with the notion of a Gr$\acute{o}$bner Basis. So, our work is also an extension of this notion for a complicated ordering of monomials.