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Title: Gabor analysis for rational functions
(joint work with Alexei Kulikov and Yurii Lyubarskii)

Abstract:
Let g be a function in $L^2(\mathbb{R})$. By G_Λ, $\Lambda \subset R^2$, we denote the system of time-frequency shifts of g, $G_\Lambda = \{e^{2\pi i\omega x} g(x-t)\}_{(t,\omega)\in\Lambda}$.

A typical model set Λ is the rectangular lattice $\Lambda_{\alpha,\beta} := \alpha\mathbb{Z} \times \beta\mathbb{Z}$ and one of the basic problems of the Gabor analysis is the description of the frame set of g i.e., all pairs α, β such that $G_{\Lambda_{\alpha,\beta}}$ is a frame in $L^2(\mathbb{R})$.

It follows from the general theory that $\alpha\beta \le 1$ is a necessary condition (we assume $\alpha, \beta > 0$, of course). Do all such α, β belong to the frame set of g?

Up to 2011 only few such functions g (up to translation, modulation, dilation and Fourier transform) were known. In 2011 K. Grochenig and J. Stockler extended this class by including the totally positive functions of finite type (uncountable family yet depending on finite number of parameters) and later added the Gaussian finite type totally positive functions.

We suggest another approach to the problem and prove that all Herglotz rational functions with imaginary poles also belong to this class. This approach also gives new results for general rational functions.