Fourier interpolation and time-frequency localization

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In their recent breakthrough paper D. Radchenko and M. Viazovska proved that every Schwartz function f can be recovered from the values of it and its Fourier transform at $\pm\sqrt{n}$ by means of an interpolation formula

$$f(x) = \sum_{n=0}^{\infty} a_n(x)f(\sqrt{n}) + b_n(x)f(-\sqrt{n}) + c_n(x)\hat{f}(\sqrt{n}) + d_n(x)\hat{f}(-\sqrt{n}).$$

If we consider the corresponding interpolation sets $\Lambda = M = \{\pm\sqrt{n}\}$ and their counting functions $n_\Lambda(R) = |\Lambda \cap [-R, R]|$, we can easily see that $n_\Lambda(W) + n_M(T) \geq 4WT - O(1)$, which perfectly matches famous Slepian's $4WT$ Theorem.

Recently, we showed (A. Kulikov, arXiv:2005.12836) that a similar bound

$$n_\Lambda(W) + n_M(T) \geq 4WT - O(\log^{2+\varepsilon}(4WT))$$

holds for all such Fourier interpolation formulas. The proof is based on the properties of the so-called prolate spheroidal wave functions, which are interesting in their own right.