Eric Mortenson Todd

“Mock theta functions, continued fractions, and orthogonality”

Abstract

In light of Zwegers (2002) breakthrough work on Ramanujan's last brilliant creation---the mock theta functions, the functions have been shown to play crucial roles in number theory, Lie theory, conformal field theory, moonshine, and black hole physics. Using recent work of the Hickerson and the Mortenson (2017), where we established the interchangeable nature of multiple forms of mock theta functions, we will take mock theta functions in a new direction and determine the role they play in Gupta and Masson's q-generalization of the Askey--Wilson framework for orthogonal polynomials. In so doing, we will develop their relationships with continued fractions, biorthogonal relations, and symmetry groups.

We will investigate the following research questions:

(Q1) Determine the relationships between classical number theoretic results on sums of squares, Kronecker-type identities, generalized Ramanujan 1psi1 sums, and root systems.

(Q2) Determine the relationships mock theta functions have with orthogonal polynomials, continued fractions, and symmetry groups.

(Q3) Determine a duality analogous to the Andrews duality for Rogers--Ramanujan identities with respect to Baxter's solution to the hard hexagon model of statistical mechanics, but here for Ramanujan's tenth-order mock theta function identities and their corresponding partial theta function identities.

(Q4) Develop algorithms to evaluate suitably nice Hecke-type triple-sums in terms of Appell--Lerch functions and partial theta functions or perhaps new buildings blocks.

(Q5) Find a framework of Rogers-Ramanujan type identities, where the infinite products arise as specialized characters of N th tensor powers of modules of affine Lie algebras and to study their asymptotics.

(Q6) Calculate branching coefficients and string functions for the tensor product of irreducible representations of affine $\hat{sl}_2$

Team members include Olga Postnova (PostDoc) and Dmitry Solovyev (Student).