Abstract: The Hopf lemma, known also as the"boundary point principle", is one of the important tools in qualitative analysis of partial differential equations. This lemma states that a supersolution of a partial differential equation with a minimum value at a boundary point, must increase linearly away from its boundary minimum provided the boundary is smooth enough.

For general operators of non-divergence type with bounded measurable coefficients this result was established in elliptic case independently by E. Hopf and O. Oleinik (1952) and in parabolic case by L. Nirenberg (1953).

The first result for elliptic equations with divergence structure was proved by R. Finn and D. Gilbarg (1957). Later the efforts of many mathematicians were aimed at the extension of the classes of admissible operators and at the reduction of the boundary smoothness. We present several versions of the Hopf lemma for general elliptic and parabolic equations in divergence and non-divergence forms under the sharp requirements on the coefficients of equations and on the boundary of a domain. Also we provide a new sharp counterexample.

The talk is based on results obtained in collaboration with Alexander Nazarov.