

Stationary Solutions of Vlasov-Poisson System and Plasma Confinement in Fusion Reactors

Alexander L. Skubachevskii^a, Yulia O. Belyaeva^a, Bjorn Gebhard^b

a) RUDN University, Mikluhko–Maklaya str. 6, Moscow, Russia

b) Universität Leipzig, Augustusplatz 10, 04109 Leipzig, Germany

* E-mail of presenting person: skub@lector.ru

In this lecture we consider the Vlasov-Poisson system for two-component high-temperature plasma with external magnetic field in a three-dimensional torus. The Vlasov-Poisson system of equations regarding to density distribution functions of charged particles and electric potential describes the kinetics of high-temperature plasma in a fusion reactor. If a considerable part of particles reaches the boundary, this can lead either to destruction of the reactor, or to cooling the plasma due to its contact with the reactor wall. Therefore, it is necessary to provide plasma confinement at some distance from the vacuum container wall. In most models of thermonuclear fusion reactors an external magnetic field is used as a control ensuring plasma confinement. From the point of view of differential equations this means that one has to prove the existence of solutions of the Vlasov-Poisson system with external magnetic field for which the supports of density distribution functions do not intersect with the boundary.

In this lecture we consider the Vlasov-Poisson system for two-component high-temperature plasma with external magnetic field in the following two cases: three-dimensional torus, which corresponds to “tokamak”, and three-dimensional cylinder, which corresponds to “mirror trap”. We prove the existence of stationary solutions of the Vlasov-Poisson system in the above mentioned domains with compactly supported density distribution functions. Using symmetries of the domains, we reduce this problem to semilinear second order elliptic differential equation, which is studied with the help of sub- and supersolutions method. A construction of stationary solution is also based on the method of truncation functions.

For the first time, stationary solutions of the Vlasov-Poisson system with homogeneous external magnetic field having supports of density distribution functions strictly inside infinite cylinder were studied in [1]. The above mentioned results concerning stationary solutions with compact supports in a torus were obtained in [2].

The first and the second authors are supported by the Russian Foundation for Basic Research, grant 20–01–00288. The third author is supported by the German-Russian Interdisciplinary Science Center (G-RISC), project numbers: M-2018b-2, A-2019b-5_d.

References

[1] A.L.Skubachevskii, Vlasov–Poisson equations for a two-component plasma in a homogeneous magnetic field\\ Russian Math.Surveys **69**:2, 291-330 (2014).

[2] Yulia O. Belyaeva, Bjorn Gebhard, Alexander L. Skubachevskii, A general way to confined stationary Vlasov-Poisson plasma configurations\\ Kinetics and Related Models (2020), doi: [10.3934/krm.2021004](https://doi.org/10.3934/krm.2021004)