



## COLLOQUIUM

**Thursday, March 11, 17:15, Zoom channel 958-115-833**



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### **Binary Quadratic Forms in Prime Variables**

Let  $r(n)$  be the function that counts the number of ways to represent a natural number  $n$  as a sum of two positive squares. The number of solutions to  $a^2 + b^2 = c^2 + d^2 \leq x$ , hence, the second moment of  $r(n)$ , in integers is well-known, while if one restricts all the variables to primes Erdős showed that only the diagonal solutions, namely, the ones with  $a = c, b = d$  contribute to the main term, hence there is a paucity of the off-diagonal solutions. In this talk we discuss the second moments of  $r(n)$ , when some of the four variables are restricted to primes. In particular, we study the paucity phenomenon of the off-diagonal solutions in such problems. These mean values are not in a scope of the classical approaches in analytic number theory, i.e. the celebrated circle method, since the major arcs are too small due to the primality of certain variables, hence other tools shall be developed. The methods are largely based on Hooley's technique to tackle (on GRH) the Hardy-Littlewood problem about the representations  $N = p + a^2 + b^2$ , where  $p$  is a prime and  $a, b$  are integers, some related works of Plaksin (based on the unconditional resolution of the Hardy-Littlewood problem by Linnik), twists of Hooley delta function (which caused the final resolution of the Manin conjecture for Châtelet surfaces) and, lastly, more recent results of S. Daniel.

Everyone is welcome!