STOCHASTIC CALCULUS

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The course is devoted to stochastic calculus, which foundations were established by K. Itô. At the initial stage of the creation of this theory, it was almost impossible to foresee how fruitful it becomes. Its role in the theory of stochastic processes can be compared with the role of differential calculus in Mathematical Analysis or other disciplines. The theory of stochastic differential equations is the natural development of the theory of ordinary differential equations. Stochastic integrals with respect to Brownian motion, which sample paths have unbounded variation, are fundamentally different from the classical integrals. This difference leads to the fact that the stochastic differentials of superpositions of smooth functions with the solutions of the stochastic differential equations depend on the second derivatives of functions under differentiation and this is absolutely impossible in the classical analysis. This difference also shows that descriptions of some physical phenomena, having a constructive nature and including stochastic interactions, are based on the second order differential equations. An important example is the hit equation.

Diffusion processes generalize the Brownian motion in a natural way. The necessity of study of the diffusion processes was probably realized by physicists earlier than it came to mathematicians. A striking example of this is the equation of Einstein–Smoluchowski, describing the motion of a light particle in a viscous fluid. On the one hand, the random motion of fluid molecules interacting via collisions makes the particle move randomly, and, on the other hand, the viscosity restricts the speed of the movement. These two factors had a significant impact on discovering of the Einstein–Smoluchowski stochastic differential equation. A rigorous mathematical definition of diffusion processes was given by A. N. Kolmogorov. After the appearance of Itô's stochastic calculus, it was shown that under certain assumptions the diffusion processes defined by Kolmogorov are the solutions of the corresponding stochastic differential equations.

In the course we plan to consider the main notions and results of stochastic calculus and also we give some applications.