On chromatic numbers of parametrized subsets of R^n

The talk is devoted to a class of problems related to the classical Nelson-Hadwiger-Erdős problem on chromatic number of space. The latter term means the chromatic number of a graph whose vertices are points of space and whose edges connecting points at a unit Euclidean distance apart. The chromatic number of an arbitrary $X \subset R^n$ refers to the chromatic number of the corresponding subgraph.

Suppose that for some sequence of sets converging to the set X in the sense of the Hausdorff metric, the lower estimate of the chromatic number that can be obtained is strictly larger than that for X itself. The simplest example is a circle of a radius tending to $1 \over 2$. A number of nontrivial problems of this kind arise when considering neighborhoods of a linear manifold in higher dimensional space, as well as chromatic numbers of spheres and neighborhoods of spheres in small dimensions. Topological methods going back to the work of L. Lovasz, direct geometrical constructions, and computer calculations are used to prove the results.