Winter School-Conference

Analysis, Geometry and Mathematical Physics – 2021

PROGRAM

December 13

15-00 - 16-00 Anton Zorich, ***Count of meanders on surfaces of higher genera***
Coffee-break
16-20 - 17-20 Nicola Arcozzi, ***A biased survey of dyadic potential theory***
17-30 - 18-30 Alexander Borichev, ***Approximation by the simplest fractions in the Bergman spaces***

December 14

14-30 - 15-30 Anton Baranov, ***Spectral synthesis for systems of exponentials and reproducing kernels***
15-40 - 16-40 Ari Laptev, ***From Weyl asymptotics to Lieb-Thirring Inequalities***
Coffee-break
17-10 - 18-10 Marco Bertola, ***Character varieties, shear coordinates and cluster algebras***
18-20 - 19-20 Dmitry Korotkin, ***Isomonodromic deformations, Goldman bracket and Fock-Goncharov coordinates***

December 15

14-30 - 15-30 Galina Lazareva, ***Mathematical modeling of melt rotation during pulsed heating of a metal target***
15-40 - 16-40 Nikita Rastegaev, ***Dissipation ratio dependent vanishing viscosity solutions of the Riemann problem for the conservation laws system representing chemical flooding***
Coffee-break
17-10 - 18-10 Nikolai Filonov, ***On the spectrum of differential operators with periodic coefficients***
18-20 - 19-20 Leon Takhtajan, ***Character varieties and moduli spaces***

ABSTRACTS

Nicola Arcozzi (University of Bologna), ***A biased survey of dyadic potential theory***

This is a (very partial) survey of concepts and phenomena in potential theory on trees and related structures; with some selected applications to combinatorics, function spaces, classical potential theory, and more. Some open problems will be mentioned.

Anton Baranov (Saint Petersburg State University), ***Spectral synthesis for systems of exponentials and reproducing kernels***

A complete and minimal system of vectors in a Hilbert space is said to admit spectral synthesis if any vector can be approximated in the norm by linear combinations of partial sums of the Fourier series with respect to this system. It was a long-standing problem whether any complete and minimal system of exponentials in $L^2(-a,a)$ admits spectral synthesis. Several years ago jointly with Yu. Belov and A. Borichev we gave a negative answer to this question which implies, in particular, that there exist non-harmonic Fourier series which do not admit a linear summation method. At the same time we showed that any exponential system admits the synthesis up to a one-dimensional defect. In the talk we will also discuss related problems for systems of reproducing kernels in Hilbert spaces of entire functions (such as Paley-Wiener or Fock).

Marco Bertola (SISSA and Concordia University), ***Character varieties, shear coordinates and cluster algebras***

In conjunction with the talk of Dmitry Korotkin, I will explain how to explicitly provide a simple (i.e.: rational) parametrization of character varieties in terms of shear coordinates (Thurston/Fock), which are a precursor to the nowadays ubiquitous "Fock-Goncharov" coordinates.

We will thus focus on 2x2 matrices (SL_2 or PSL_2). The shear coordinates have practical advantages since they allow to write the Goldman Poisson structure in explicit and simple form. If time permits, I will explain why cluster algebras come into play (and what they are).

Alexander Borichev (Aix-Marseille University), ***Approximation by the simplest fractions in the Bergman spaces***

We discuss several questions related to the approximation by finite sums of the Cauchy kernels in weighted Bergman spaces. In particular, we deal with the Chui conjecture in the Hilbert space situation.

Nikolai Filonov (POMI and Saint Petersburg State University), ***On the spectrum of differential operators with periodic coefficients***

We consider the Schrödinger operator $H = -D + V(x)$ in the space $L^2(\mathbf{R}^d)$. Here V is a real-valued function which is periodic with respect to a lattice in $\mathbf{R}^d$. The operator H is self-adjoint and semi-bounded from below; its spectrum lies on the real line. The need to study such operators and their spectra naturally arises in solid state physics. It is known that the spectrum has the band-gap structure, i.e. it is a countable union of intervals (bands) which can overlap. We discuss the question of whether there can be degenerate spectral bands, which means that the corresponding intervals collapse to a point, and the spectrum gets an eigenvalue of infinite multiplicity.

Dmitry Korotkin (Concordia University and CRM), ***Isomonodromic deformations, Goldman bracket and Fock-Goncharov coordinates***

The goal of the lecture is to give an introduction into three different but intertwined topics. The first is the classical theory of isomonodromic deformations of linear systems of differential equations of a complex variable. The equations governing these deformations were discovered by Schlesinger about 100 years ago; they include all known special functions as elementary special cases. The theory of Schlesinger equations attracted a lot of attention again in 1980's wen its close connection to the quantum field theory was revealed in works of Jimbo, Miwa, Sato and others, where, in particular, it was introduced the isomonodromic tau-function which quickly occupied the central place in the theory. Later symplectic aspects of the monodromy map were studied; it turned out that the monodromy map is Poisson with respect to the natural choice of the Poisson brackets on the source (Kirillov-Kostant) and target (Goldman) spaces. More recently, it became clear that the Jimbo-Miwa tau-function can be naturally defined as the generating function (in the sense of symplectic geometry) of the monodromy map. To make this definition computationally efficient one needs to use the celebrated Fock-Goncharov coordinates on the monodromy manifold; this leads to rather non-trivial results even in application to the standard hypergeometric equation.

Ari Laptev (Imperial College London and Saint Petersburg State University), ***From Weyl asymptotics to Lieb-Thirring Inequalities***

We shall present the state of art of the area of Spectral Theory of Partial Differential Operators related to Lieb-Thirring inequalities as a natural development of Weyl’s type asymptotic formulae.

Galina Lazareva (Peoples' Friendship University, Moscow), ***Mathematical modeling of melt rotation during pulsed heating of a metal target***

On the experimental stand Beam of Electrons for materials Test Applications (BETA) created in the INP SB RAS, the results of heating the tungsten plate by the action of a high-speed electron beam on it were obtained. We consider an extended dynamic model of current distribution when the surface of a tungsten sample is heated by an electronic beam pulse. The temperature in the sample, calculated from the two-phase Stefan problem, is needed to solve the electrodynamic equations. A special case of axial symmetry without taking into account electrical driving forces is considered. The current is considered as a possible source of rotation of the substance, which is observed in the experiment. The aim of the study is to simulate the erosion of the sample surface as a result of evaporation and penetration of heat flux into the material.

Nikita Rastegaev (POMI and Saint Petersburg State University), ***Dissipation ratio dependent vanishing viscosity solutions of the Riemann problem for the conservation laws system representing chemical flooding***

We study the solutions of the Riemann problem arising in chemical flooding models. Vanishing viscosity admissibility criterion is used to distinguish physically meaningful weak solutions. We demonstrate that when the flow function depends non-monotonically on the chemical agent concentration, non-classical undercompressive shocks appear. We prove the monotonic dependence of the shock velocity on the ratio of dissipative coefficients. For that purpose we classify the phase portraits for the travelling wave dynamical systems and study the saddle-saddle connections.

Leon Takhtajan (Stony Brook University), ***Character varieties and moduli spaces***

I will review basic facts about symplectic structures on character varieties and moduli spaces of algebraic curves and vector bundles, and relations between them.

Anton Zorich (Skoltech and University of Paris), ***Count of meanders on surfaces of higher genera***

I will introduce square-tiled surfaces and will explain why they represent integer points in the moduli spaces of Abelian and quadratic differentials. I will also give a hint on why the count of square-tiled surfaces provides Masur-Veech volumes of these moduli spaces. I will show how count of square-tiled surfaces allows to count meanders and orientable meanders on surfaces of genera higher than zero. The talk is based on a joint work with V. Delecroix, E. Goujard and P. Zograf.