# Kružkov-type uniqueness theorem for the chemical flood conservation law system

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We consider the system

$$\begin{cases} s_t + f(s,c)_x = 0, \\ (cs + a(c))_t + (cf(s,c))_x = 0, \end{cases}$$

most commonly used to describe the flood of the oil reservoir with a chemical solution. The flow function $f$ is commonly assumed to be S-shaped in $s$. The adsorption function $a$ is often concave and usually represented by the Langmuir adsorption isotherm. In our work, we limit ourselves to functions $f$ monotone in $c$.

This system is neither strictly hyperbolic nor genuinely non-linear, therefore known results for strictly hyperbolic genuinely non-linear systems of conservation laws are not directly applicable.

The solutions for some boundary-initial problems for this system were explored, for example, in [1] (Riemann problem), [2] and [3] (slug injection). The last two papers use the Lagrange coordinate transformation to split the equations and the characteristics method to construct solutions. However, the question of the uniqueness of the constructed solutions is not covered.

We use the proposed coordinate change to prove a Kružkov-type uniqueness theorem for the Cauchy problem with several limitations on the initial data and the class of weak solutions under consideration. The vanishing viscosity method is utilized to determine admissible shocks.

This talk is based on the joint work with S. G. Matveenko.

## References

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