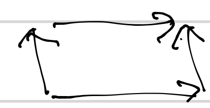
Lecture 01

1. Riemann surface — 1D analytic manifold:

   - Definition of manifold
   - Local maps
   - Complex structure

2. Examples $\mathbb{C}$, $\overline{\mathbb{C}}$, $\mathbb{T}^2$

   Realization $\mathbb{T}^2$ as 

   How do we choose coordinates when gluing.

3. Compacteness

4. Holomorphic, meromorphic functions $M \to \mathbb{C}$, $M \to N$:

   a. Types of isolated singularities
   b. Domain preservation
   c. Maximum principle

Exercises.

   d. Liouville theorem
   e. $f: M \to N$, M - compact $\Rightarrow$ either $f(M) = N$ or $f = \text{const}$.

5. More examples:

   a. Meromorphic functions on $\mathbb{T}^2$ = = two periodic meromorphic functions on $\mathbb{C}$

   b. Holomorphic functions $\overline{\mathbb{C}} \to \mathbb{C}$ - rational functions

10. We study "rational functions" on compact analytic surfaces.

11. List of properties of rational functions (to mimique).

12. Exercise. There is no meromorphic f-n on T^2 with one simple pole.

13. $f: M \to N$; multiplicity, branching #. (local # of preimages).

14. Proposition $f: M \to N$, M, N - compact

$$m = \sum_{P \in f^{-1}(Q)} b_f(P) + 1$$

is independent of $Q \in M$

15. Degree of mapping.

16. Covering mapping.

17. Reminder about real-valued topology

- Real manifolds, smooth functions etc
- Simplex, oriented simplex
- Triangulation
- Orientable manifolds: two definitions

Ex: Prove equivalence

- Example Projective plane
- Exercise Sketch a triangulation of projective plane.

18. Riemann surface is an orientable 2D real manifolds.