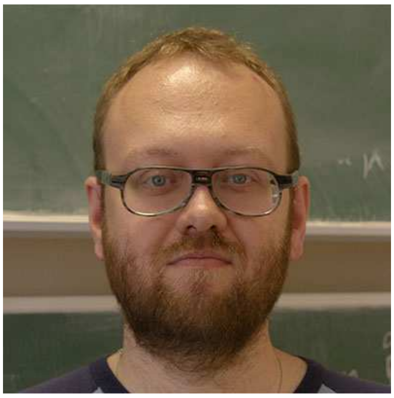
Факультет математики и компьютерных наук
Санкт-Петербургский государственный университет

КОЛЛОКВИУМ

четверг 8 октября 17:15, канал Zoom 675-315-555



Fedor Petrov (M&CS SPbU)

$\boldsymbol{A + \sqrt[2]{2}A}$

Let A be a finite subset of $\boldsymbol{R}$ of size $|A| = n$, and λ be a real number. How small can be the set $A + \lambda A = \{a + \lambda b : a, b \in A\}$? A related continuous question: how small can $K + T(K)$ be if $K \subset \boldsymbol{R}^d$ is a compact set of Lebesgue measure 1 and T is a given linear operator. These topics were studied by Bukh, Konyagin and Łaba, Balog and Shakan, Sanders, Schoen, Chen and Fang, Mugdal and others. When $\lambda = p/q$ is a primitive rational fraction, the lower bound is known to be $|A + \lambda A| \geq (|p| + |q|)n - C(p, q)$; if λ is transcendental, it becomes superlinear: $|A + \lambda A| \geq \exp(\log^c n) \cdot n$ for a universal constant $c > 0$.

We find the sharp constant for the simplest algebraic but irrational number $\lambda = \sqrt[2]{2}$; also we answer the above continuous question and discuss the further things to think about. Our proofs combine Freiman's structural theorem for the sets with small sumsets and the discrete and continuous versions of Brunn-–Minkowski and Prékopa–Leindler inequalities.

The talk is based on a joint work with Dmitry Krachun.

Приглашаются все желающие!