QUANTITATIVE ASPECTS OF NORMAL GENERATION OF $\mathrm{SL}_2(R)$

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It has been known by work of Carter-Keller and Tavgen since the 90s that split Chevalley groups $G(\Phi, R) =: G$ defined using rings R of S-algebraic integers and irreducible root systems Φ of rank two are boundedly generated by root elements. Work by Kedra-Gal has further shown that if a finite collection of conjugacy classes generates $G(\Phi, R)$, then it boundedly generates $G(\Phi, R)$. Also, it was shown in the case of $G = \mathrm{SL}_n(R)$ for $n \geq 3$ by Morris that there is a bound (for bounded generation) only depending on the number of finitely many conjugacy classes (rather than the classes themselves) that are taken as a generating set and by Kedra-Libman-Martin that the bound is actually linear in the number of conjugacy classes, if R is a principal ideal domain. A group with this property is called *strongly bounded.* In this talk, I will explain a method to generalize strong boundedness results to other $G(\Phi, R)$ for arbitrary rings of algebraic integers and all split Chevalley groups groups by using Gödels Compactness theorem together with classical Sandwich Classification Theorems of split Chevalley groups. I will demonstrate this method in the case of $\mathrm{SL}_2(R)$ for R a ring of S-algebraic integers with infinitely many units. I will also, if time allows, talk about the existence of small normally generating subsets of $G(\Phi, R)$ and explain how the existence or non-existence of small normally generating sets distinguish $\mathrm{Sp}_4(R), G_2(R)$ and $\mathrm{SL}_2(R)$ from the other $G(\Phi, R)$ in regards to strong boundedness.

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