Flips in order-k Delaunay triangulations

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Abstract

Consider a set S of n points in the plane. A triangulation of set S is called the *Delaunay* triangulaiton of S if for every triangle its circumcirce does not enclose (i.e., contain in its interior) any point of S. In this talk we mainly focus on the following generalization of this well-known object. A triangulation of set S is an *order-k Delaunay* triangulation of S if for every triangle its circumcircle encloses at most k points of S.

When studying triangulations, a very useful structure is the *flip graph* and its variations. The flip graph of S has one vertex for each possible triangulation of S, and an edge connecting two vertices when the two corresponding triangulations can be transformed into one another by a *flip* (i.e., exchanging the diagonal of a convex quadrilateral by the other one). The flip graph is an essential structure when studying triangulations. For example, any triangulation of S is connected with the Delaunay triangulation of S by a path of length $O(n^2)$ in the flip graph, and there are examples where a quadratic number of flips is necessary.

The flip graph as restricted to order-k Delaunay triangulations of S might be disconnected for $k \geq 3$. However, any order-k triangulation can be transformed into some other order-k triangulation by at most $k-1$ flips, such that the intermediate triangulations are order-$(2k-2)$ triangulations, if S is a set of points in convex position. We will discuss these results and see some illustrations.

No prior knowledge beyond the first-year university program is required.

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