On subgroups of linear groups over a ring

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Let $G = SL_n$ – be a special linear group, $n \geq 3$, and let K be a subring of a commutative ring A with 1. It is a long standing problem to classify the subgroups of $G(A)$, contaning the elementary subgroup $E(K)$. It turns out that in the standard situation all such subgroups are closed to $G(R)$ over an intermediate subring R. More precisely, for a subgroup H: $E(K) \leq H \leq G(A)$ there exists a unique subring R of A such that $E(R) \leq H \leq N_A(R))$, where $N_A(R)$ denotes the normalizer of $E(R)$ in $G(A)$. Besides, the quotient group $N_A(R)/E(R)$ is soluble for finite dimensional rings and well understood for rings of Krull dimension ≤ 1.

Instead of SL_n one can poses the same question for a split classical group or, more generally, a Chevalley group G. In the talk we present a condition on the pair of rings and the type of a Chevalley group that conjecturally equivalent to the standard description of the subgroup lattice between $E(K)$ and $G(A)$, formulate all known results, and discuss an idea of the proof of our conjecture.