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Triangulated categories and weight structures

The talk can be interesting to anyone who had some experience with (co)homology and categories; I will recall definitions that are necessary for understanding (during the talk).

One of the main ideas of homological algebra is to study (co)homology to obtain functors with values in "more interesting categories". These are often *triangulated*. A basic example of a triangulated category is the homotopy category of complexes (say, of R-modules, where R is a ring). Objects of $K(R)$ are complexes of R-modules, morphisms are sequences of homomorphisms between terms "compatible with the differentials", and one has to identify homotopic sequences. Complexes can be shifted ("to the left"); $C[1]^i = C^{i+1}$. One often have to make the morphisms of complexes that give isomorphisms on cohomology invertible; this is a formal construction that gives the *derived category* $D(R)$. Its definition is rather nice; yet explicit computations may be difficult. To $C \in D(R)$ one can clearly associate its cohomology modules $H^i(C)$; yet more (functorial) information is available.

One can pass from C to $H^0(C)$ into two steps: one can kill cohomology in positive degrees first and kill negative degree one after that. These steps are achieved by means of *canonical truncations* of complexes; a generalization of those gives *t-structures* on triangulated categories. These truncations are functorial and enable to "cut C into its cohomology modules"; these pieces are certainly "much simpler than C".

It is even easier to define *stupid truncations*: just kill the terms C^i in all non-negative or in non-positive degrees. The corresponding pieces of C are just C^i; yet they are not canonical (and functorial). I defined *weight structures* that allow to deal with this problem. A key role is played by *connectivity* conditions. Since there exist only trivial extensions (of positive degrees) between free modules ($\cong \bigoplus_I R$; this is also true for their direct summands, that is, *projective* modules), there exists a "projective" weight structure on $D(R)$; it corresponds to projective resolutions. Since negative stable homotopy groups of a point are zero, there exists a (spherical) weight structure on the stable homotopy category SH (of spectra). The latter gives cellular filtrations on spectra; note that a morphism of spectra extends to a morphism of their cellular filtrations but these extensions are not unique.

(Chow) weight structures yield functorial Deligne-type weight filtrations on any (co)homology of Voevodsky motives; whence the name. One obtains "functoriality up to homotopy" of nice compactifications of smooth varieties.