On the impact of diffusion ratio on vanishing viscosity solutions of Riemann problems for chemical flooding models

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We will discuss the vanishing viscosity solutions for a chemical flooding model ($x \in \mathbb{R}$, $t \in \mathbb{R}_+$) as $\varepsilon_c, \varepsilon_d \to 0$:

$$\begin{gathered} s_t + f(s,c)_x = \varepsilon_c (A(s,c) s_x)_x, \\ (cs + a(c))_t + (cf(s,c))_x = \varepsilon_c (cA(s,c)s_x)_x + \varepsilon_d c_{xx}. \end{gathered} \tag{1}$$

Here $s = s(x,t)$ is the water phase saturation, $c = c(x,t)$ is the concentration of the chemical agent in the water phase; the function f denotes the fractional flow of water; the function a denotes the chemical's adsorption on the rock. It is commonly assumed that f is an S-shaped function of s for every c, and a is an increasing concave function. This system is often used to describe the displacement of oil by a hydrodynamically active chemical agent (polymer, surfactant, etc).

The main message of the talk is the following:

- if $f(s,c)$ is monotone with respect to c, then there exists a unique vanishing viscosity solution to the Riemann problem (see [1])
- if $f(s,c)$ is non-monotone with respect to c, then there may exist *not unique* vanishing viscosity solution to the Riemann problem. In particular the limit depends on the ratio of diffusion parameters $k = \varepsilon_d/\varepsilon_c$. Some examples are given in [2], see also [3]. We formalize and generalize these results. The key object of our study is the travelling wave between two saddle points of the travelling wave dynamical system corresponding to (1).

This is a joint work with F. Bakharev, A. Enin, N. Rastegaev.

References

[1] Johansen T. and Winther R. The solution of the Riemann problem for a hyperbolic system of conservation laws modeling polymer flooding // SIAM journal on mathematical analysis. — 1988. — Vol. 19, no. 3. — P. 541–566.

[2] Shen W. On the uniqueness of vanishing viscosity solutions for Riemann problems for polymer flooding // Nonlinear Differential Equations and Applications NoDEA. — 2017. — Vol. 24, no. 4. — P. 37.

[3] Entov V. and Kerimov Z. Displacement of oil by an active solution with a nonmonotonic effect on the flow distribution function // Fluid Dynamics. — 1986. — Vol. 21, no. 1. — P. 64–70.

Everyone is welcome!