

Non-real zeroes of homogeneous differential polynomials and generalisations of the Hawaii conjecture

by

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Given a real polynomial $p(z)$ with only real zeroes, we estimate the number of non-real zeroes of the differential polynomial

$$F_{\varkappa}[p](z) = p(z)p''(z) - \varkappa[p'(z)]^2,$$

where \varkappa is a real number.

A counterexample to a conjecture by B. Shapiro on the number of real zeroes of the polynomial $F_{\frac{n-1}{n}}[p](z)$ in the case when the real polynomial $p(z)$ of degree n has non-real zeroes is constructed.

There will be presented some new approaches promising to simplify the proof of the Hawaii conjecture, and the general situation with the Laguerre and Newton inequalities for real polynomials and their generalisations will be discussed. We will also discuss other generalisations of the Hawaii conjecture and possible extensions of our result to entire functions.

The talk is based on joint work with Mohamed J. Atia, Olga Katkova, and Anna Vishnyakova.