APPLICATIONS OF CONICS IN THE THEORY OF QUADRATIC FORMS AND CENTRAL SIMPLE ALGEBRAS

We discuss several interrelated problems from the theory of quadratic forms and central simple algebras over fields. Their proofs have a common feature, namely, one of the main tools in the argument is a conic and its function field.

• We construct an indecomposable division algebra in the relative Brauer group ${}_2\operatorname{Br} F(\sqrt{a_1}, \sqrt{a_2} \dots \sqrt{a_n})/F$, where k is a field, $n \geq 3$, $a_1, \dots, a_n \in k^*$ are given elements, and F/k is a suitable field extension. This problem is related to non-excellence of 2-primary field extensions and the homology groups of the Brauer complex for a triquadratic extension, which are of their own interest, and will be also discussed.

• We consider the extension $k(X_1 \times X_2)/k$, where X_i is the conic corresponding to the quaternion algebra Q_i over a field k. We prove that this extension becomes nonexcellent after replacement of the ground field k by a suitable extension F, provided that $\operatorname{ind}(Q_1 \otimes_k Q_2) = 4$. This result permits to show that the torsion of the Chow group $CH^2(X_1 \times X_2 \times X_3)$ over F equals $\mathbb{Z}/2\mathbb{Z}$ for a suitable conic X_3 over F such that $\operatorname{ind}(Q_1 \otimes_k Q_2 \otimes_k Q_3) = 4$. This means that there is no such an extension L/F of degree $4m$ with an odd m that all three conics X_{iL} split.

• The method in the previous problem permits to construct biquaternion algebras D_1 and D_2 over a field k without a common biquadratic splitting field with $\operatorname{ind}(D_1 \otimes_k D_2) = 2$.

• Let C_1, C_2, C_3 be conics with the corresponding quaternion algebras Q_i over a field F. Assume that $\operatorname{ind} \alpha \leq 2$ for any α in the subgroup of ${}_2\operatorname{Br}(F)$ generated by all Q_i. We give a necessary condition on the level of quadratic forms for triviality of the torsion of $CH^2(C_1 \times C_2 \times C_3)$, which is equivalent to existence of a common slot for algebras Q_i.

• Applying the excellence property of conics, we prove that for any quadratic forms φ_1 and φ_2 over a field F, and $d \in F^*$, the anisotropic part of the form $\varphi_1 \perp (t^2 - d)\varphi_2$ over $F(t)$ has a similar type, i.e. there are forms τ_1 and τ_2 over F such that $(\varphi_1 \perp (t^2 - d)\varphi_2)_{an} \simeq \tau_1 \perp (t^2 - d)\tau_2$. Further, let φ be a 4-dimensional form over F, $a, b \in F^*$. Using the same method, we give a criterion for the anisotropic part of the form $\varphi_{F(\sqrt{a},\sqrt{b})}$ to be defined over F.

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