Упражнения на асимптотику зависящих от параметра интегралов

1.11.22

При выполнении заданий не разрешается пользоваться готовыми формулами для асимптотик.

Список литературы

[St] E.M. Stein, *Harmonic Analysis: Real-Variable Methods, Orthogonality, and Oscillatory Integrals*, Princeton University Press, 1993.

[Fe] М.В. Федорюк, *Метод перевала*, Наука, М., 1977.

[Zw] M. Zworski, *Semiclassical Analysis*, Graduate Studies in Mathematics **138**, American Mathematical Society, Providence, RI, 2012, `http://www.ams.org/books/gsm/138/`.

Метод Лапласа

1. $$\int\limits_{\mathbb{R}} \frac{dx}{(1+x^2)^n}.$$

2. $$\int\limits_{-1}^{1} (2+x^2-x^4)^n(2x^2-1).$$

3. $$\int\limits_{-\sqrt{2}}^{\sqrt{2}} (1+x-|x|)(2x^2-x^4)^n.$$

4. $$\int\limits_{\mathbb{R}} \frac{\sin^n x dx}{x^n}.$$

5. $$\int\limits_{-1}^{1} (239-\sqrt{|x|}-\sqrt[3]{|x|})^n\, dx.$$

6.
$$\int_{\mathbb{R}} \frac{dx}{(x^2+n)^n}.$$

7.
$$\int_0^1 (1 - x^4 + 3x^5 - 3x^6 + x^7)^n \, dx.$$

8.
$$\int_{\sum_{j=1}^n x_j^2 \le 1} \cos^{\frac{2n}{\pi}} \Big(\sum_{j=1}^n x_j^2 \Big) \, dx_1 \, dx_2 \ldots \, dx_n.$$

9.
$$\int_{\mathbb{R}} \frac{\cos^n(\pi x) \, dx}{1+x^2}.$$

10.
$$\operatorname{vp} \int_{\mathbb{R}} \frac{e^{-n(x-1)^2}}{x} \, dx.$$

11.
$$\int_0^1 (1 + x \log x)^n \, dx.$$

12.
$$\int_{\mathbb{R}^2} (1 + |x|)^{-n} \cos x \, dx.$$

13.
$$\int_{\mathbb{R}^2} \Big(1 + \sqrt{(x-1)^2 + y^2} + \sqrt{(x+1)^2 + y^2} \Big)^{-n} \, dx \, dy$$

14.
$$\int_{x^2+y^2 \le 1} (1 - x^4 - y^4) \cos^n(1 - x^2 - y^2) \, dx \, dy$$

15.
$$\int_{\mathbb{R}} (n + |x| \cos x)^{-n} \, dx.$$

16.
$$\int_0^{\frac{\pi}{2}} x^{\frac{\pi n+1}{4}} \cos^n x \, dx$$

17.
$$\int\limits_{-2}^{2} \exp\left(\lambda\cos x - \mu(x-1)^2\right)\,dx, \quad \lambda,\mu\to+\infty,\ \mu=o(\lambda).$$

18.
$$\int\limits_{0}^{+\infty} n^{t-2\ln t}\,dt, \quad \text{найдите два слагаемых асимптотики}$$

19.
$$\int\limits_{\mathbb{R}} \exp\left(\mu\cos x - \lambda x^2\right), \quad \lambda,\mu\to+\infty,\ \mu=o(\lambda).$$

20.
$$\sum_{k=0}^{n^2} \cos^n\left(\frac{k}{n^2}\right)$$

21.
$$\sum_{j=[n/2]}^{2n} \frac{n^j}{j!}$$

22.
$$\int\limits_{0}^{1} \exp(-1/x - nx^2)\,dx$$

Метод стационарной фазы

1.
$$\int\limits_{0}^{1} \cos(nx^2(1-x)^2)\,dx.$$

2.
$$\operatorname{vp}\int\limits_{-3}^{3} \frac{\sin(n|x|^{\frac{3}{2}})}{x+x^2e^{-x}}\,dx.$$

3.
$$\int\limits_{\mathbb{R}} \frac{e^{2\pi i n\cos 2\pi x}}{1+x^2}\,dx$$

4.
$$\int\limits_{0}^{1} e^{2\pi i(n\cos 2\pi x^2+\sin 2\pi x^2)}\,dx$$

5.
$$\int_{-1}^{1} e^{2\pi i(n\sqrt{1-x^2}+x)} + e^{2\pi i(n\sqrt{1-x^2}-x)}\,dx.$$

6.
$$\int_{x^2+y^2\le 1} (1-x^4-y^4)\cos(n(1-x^2-y^2))\,dx\,dy$$

7.
$$\int_{\mathbb{R}} e^{2\pi in(1-x^4+3x^5-3x^6+x^7)}\,dx.$$

8.
$$\int_{\mathbb{R}} e^{-\frac{1}{x^2}-x^2} e^{2\pi i(nx+\frac{n}{x})}\,dx$$

9.
$$\int_{\mathbb{R}^2} \frac{e^{2\pi in\left(\sqrt{(x-1)^2+y^2}+\sqrt{(x+1)^2+y^2}\right)}}{(1+x^2+y^2)^2}\,dx\,dy.$$

10.
$$\int_{\mathbb{R}} e^{-\pi x^2-in\sqrt{|x|}-in\sqrt[3]{x}}\,dx$$

11.
$$\int_{\mathbb{R}^2} \frac{e^{\pi in(y^2-x^2)}}{(1+x^2+y^2)^2}\,dx\,dy.$$

12.
$$\operatorname{vp}\int_{\mathbb{R}} \frac{e^{2\pi inx^3}}{x-1}\,dx$$

13.
$$\int_{-1}^{1} \cos\left(x+\frac{1}{n}\right)\sin \pi n x^2\,dx$$

14.
$$\int_{\mathbb{R}} e^{-\pi x^2+in^2\cos(\frac{x^2}{n^2})}\,dx$$

15.
$$\sum_{j=-n^n}^{n^n} \cos(2\pi n^{1-2n}j^2).$$

16.
$$\sum_{j=-n^{120}}^{n^{120}} \cos(2\pi n^{-239} j^2).$$

17.
$$\int_{-2}^{2} \exp\left(i\left(\lambda\cos x - \mu(x-1)^2\right) - \frac{1}{4-x^2}\right), dx, \quad \lambda,\mu\to+\infty,\ \mu=o(\lambda).$$

18.
$$\int_{-2}^{2} \exp\left(i\left(\lambda\cos x + \mu x^2\right) - \frac{1}{4-x^2}\right)\, dx, \quad \lambda,\mu\to+\infty,\ \mu=o(\lambda).$$

19.
$$\int_{0}^{1} \ln x\cdot\exp\left(in\cos x - \frac{1}{1-x^2}\right)\, dx$$

20.
$$\int\limits_{\{x^2+y^2\le 1\}} \frac{x^2}{1+y^4}\cdot\exp\left(in\cos(x^2+y^2) - \frac{1}{1-x^2-y^2}\right)\, dx\, dy$$